EXPERIMENTAL STUDIES ON DYNAMIC RESPONSE OF STRUCTURES

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CERTIFICATE

Certified that the thesis entitled 'Experimental Studies on the Dynamic Response of Structures' is a bonafide work done by Mr. M. Sahabuddin, under our guidance and has not been submitted for the award of any degree elsewhere.

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ABSTRACT

Experimental studies on dynamic response of structures are carried out on cantilever beams, single storey and double storey portal frames. Maximum absolute displacements for different ground displacements are measured. These are compared with theoretical values and certain conclusions are drawn.

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CHAPTER 1

INTRODUCTION

The interest in the structural design for dynamic loads has been steadily increasing over the years. More importance is given to the dynamic design because of the fact that larger structures with minimum weight are being attempted in the Aircraft industry and amongst the Civil Engineering Structures. Such structures are more susceptive to dynamic effects; because, in general, they are more flexible and have larger natural period (11).

It is, therefore, necessary to seek ways and means for predicting the dynamic response of structures subject to time dependent loadings as accurately as possible. With the advent of digital computers it has been possible to analyse large size structures subjected to the dynamic loads and thus predict the response based on a preselected theory. These theories involve idealization at various stages of analysis particularly so when the structure is complex. The purpose of the present work is to experimentally verify the results obtained from analytical computation with those observed in the laboratory on models. As a first phase of the work this comparison has been performed on rather simple structures, namely, the cantilever beam and portal frames.

Note: The numbers in the parantheses refer to the reference

From this study certain conclusions are drawn which can be directly used to predict the dynamic response of complex structures based exclusively on model experimentation.

In advanced countries like U.S.A., U.K., Germany and Japan (10) the prototypes are tested under simulated load conditions to study the dynamic response of large sized structures. However, this has its own financial and technological involvements. Therefore, resort is taken to model tests. The latter is however not absolutely free of problems. The first to handle is the production of model itself which has to be dynamically equivalent to the prototype. Secondly, presuming that such a model has been produced and experiment has been conducted, it still remains to be checked whether the prototype will behave in a manner similar to the one revealed by the model. In the present work no attempt has been made to simulate the structural model. Simple structures have been considered on the laboratory scale and prototype models have been fabricated for experimentation. These structures have been analysed by the existing theories of dynamic analysis on the computer and results are compared with those obtained experimentally.

In general, the dynamic loadings (7) can be classified into three types :

(1) Transient loading

(2) Periodic loading

(3) Non-periodic loading

The transient loading occurs for a short duration. It produces transient response the classical example would be a blast.

The periodic loading is cyclic in nature and has a finite time period. If large number of cycles occur, then, the response of the structure will be in a periodic state. Example of this type of loading will be a running motor mounted on floors.

The non-periodic loading is random in nature. Therefore, the information about the random nature has to be stated in a statistical term.

In general, any loading will be random and this could be approximated to periodic and sinusoidal by using fourier expansions.

In the present work, the forcing function is assumed to be sinusoidal and throughout this work only sinusoidal forcing function.is considered.

Chapter 2 describes the mathematical model of the structure on which the present study has been conducted. The experimental model of the structure is described in Chapter 3. Chapter 4 deals with the experimental set-up. In this work, the instruments which have been used are :

1. Unholds-Dickie Vibration System
2. M.B. Vibration Exciter

3. Travelling Microscope
4. Strobotac
5. M.B. Vibration Meter

Results and conclusions are discussed in Chapter 5.

CHAPTER 2

MATHEMATICAL MODEL

2.1.0 Cantilever Beam Subjected to Sinusoidal Base Motion :

The assumptions made in setting the equations for dynamic response are well documented in the literature. Still, for the sake of completeness, they are briefly mentioned here. It is assumed that the beam cross-section remains plane before and during vibration. Only small deflections occur so that the radius of curvature is small which is approximated to d^2y/dx^2. The static modulus of elasticity is approximated to dynamic modulus. The effects of shear, rotation and damping are neglected and the system is considered to be linear and conservative.

2.1.1 The Governing Equation of Motion :

The cantilever beam shown in Figures 1 to 3 is considered as a continuous system with distributed mass.

The equation of motion is given by (1, 2, 3, 4, 5, 6)

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0 \qquad (2.1.1)$$

where E is the Young's modulus

I is the moment of inertia

m is the mass per unit length

t is the time

is the space variable

y is the displacement at any point of the cantilever

The solution of Equation 2.1.1 yields the response of the structure. The free vibration analysis requires the determination of the natural frequency and mode shape.

2.1.2 Free Vibration :

$$\text{Let } y = Y(x) \, . \, T(t) \qquad (2.1.2)$$

be the solution of the Equation 2.1.1. The Equation 2.1. becomes

$$EI \frac{Y^{iv}}{Y} + m \frac{\ddot{T}}{T} = 0 \qquad (2.1.3)$$

where prime denotes the differentiation with respect to space and the dot denotes the differentiation with respect to time.

Equation 2.1.3 can be rearranged as

$$\frac{EI}{m} \frac{Y^{iv}}{Y} = \frac{\ddot{T}}{T} \qquad (2.1.4)$$

The left hand side of Equation 2.1.4 is in terms of space variable and the right hand side is in terms of time variable. This is possible only when either side of Equation 2.1.4 is equal to a constant. Let this constant be p^4

where p is the natural frequency.

Therefore the Equation 2.1.4 can be written as

$$\frac{EI}{m}\frac{Y^{iv}}{Y} = p^4$$

This could be rewritten as

$$Y^{iv} - \frac{p^4}{b^2} Y = 0 \qquad (2.1.5)$$

where $b^2 = \frac{EI}{m}$

The solution to the fourth order differential equation 2.1.5 is

$$Y = A_1 \cos kx + A_2 \sin kx + A_3 \cosh kx + A_4 \sinh kx \qquad (2.1.6)$$

where $k = \sqrt{p^2/b}$

where the boundary condition, presuming the structure to be at the quiescent state are given by

$$y(o,t) = 0$$
$$y'(o,t) = 0$$
$$y''(l,t) = 0$$
$$y'''(l,t) = 0$$

The solution of Equation 2.1.6 contains four constants A_i, $i = 1, 2, 3, 4$. Furthermore the frequency parameter k is

also unknown. The relation between the constants A_i, $i = 1, 2, 3, 4$ and the parameter k obtained after applying the above boundary conditions are as follows :

$$A_1 + A_3 = 0$$

$$A_2 + A_4 = 0$$

$$-A_1 \cos kl - A_2 \sin kl + A_3 \cosh kl + A_4 \sinh kl = 0$$

$$A_1 \sin kl - A_2 \cos kl + A_3 \sinh kl + A_4 \cosh kl = 0$$

The above set of equations are simultaneous homogeneous equations which form the eigen value problem with k as the eigen value. The constants A_i, $i = 1, 2, 3, 4$ define the mode shape corresponding to eigen value.

Solving the above set of equations, the resulting equation is given by

$$\text{Cos } kl . \cosh kl = -1 \tag{2.1.8}$$

Equation 2.1.8 is called the frequency equation. The values of kl which satisfy the equation 2.1.8 is given in the literature of which only the first two values from the lowest end of the spectrum are taken in the present work. The natural frequencies and the corresponding mode shapes for the cantilever structure considered as shown in Figures 2 and 3 are given in Table 1a and 1b.

2.1.3 Forced Vibration :

The response of the system subjected to dynamic load consists of two parts . one is the transient response and the other is the steady state response. The transient response dies out eventually and the structure vibrates only under the steady state force input where the frequency of the forced vibration of the structure is equal to the frequency of the forcing function.

In order to obtain the steady state response, the equation of motion Equation 2.1.1 is considered again

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \qquad (2.1.1)$$

This equation is the free vibration equation and the forcing function is imposed as the boundary condition.

Let the solution to forced vibration be

$$Y \quad y = Y(x) \sin \omega t \qquad (2.1.9)$$

where y is the absolute response of the structure, Y(x) is the displacement for all time t and ω is the forcing frequency.

Substituting Equation 2.1.9 into the Equation 2.1.8, we get for all instant of time $t > 0$.

$$\frac{d^4 Y}{dx^4} - \frac{m\omega^2}{EI} Y = 0 \qquad (2.2.0)$$

The solution of equation is given by

$$Y(x) = C_1 \cos rx + C_2 \sin rx + C_3 \cosh rx + C_4 \sinh rx \tag{2.2.1}$$

where $r = \sqrt[4]{\frac{\omega^2}{EI}}$

The boundary conditions are now given by

$$y(o,t) = y_0(t) = \lambda \sin \omega t \text{ for all } t > 0$$

$$y'(o,t) = 0$$

$$y''(l,t) = 0$$

$$y'''(l,t) = 0 \tag{2.2.2}$$

substituting the above boundary conditions 2.2.2 into 2.1.9, we get

$$\lambda \sin \omega t = y(o) \sin \omega t$$

$$\text{i.e. } y(o) = \lambda$$

$$y'(o,t) = 0$$

$$y''(l,t) = 0$$

$$y'''(l,t) = 0 \tag{2.2.3}$$

With the equations 2.2.3 substituted into Equation 2.2.1 yields

$$C_1 + C_3 = \lambda$$

$$C_2 + C_4 = 0$$

$$-C_1 \cos rl - C_2 \sin rl + C_3 \cosh rl + C_4 \sinh rl = 0$$

$$C_1 \sin rl - C_2 \cos rl + C_3 \sinh rl + C_4 \cosh rl = 0$$

Above set of equations are non-homogeneous simultaneous equations which when solved yield

$$C_3 = \frac{\cos rl + (\cos rl \sinh rl + \sin rl \cosh rl) x (\sin rl + \sinh rl) / 2(\cos rl . \cosh rl + 1)}{\cosh rl + \cos rl}$$

$$C_1 = \lambda - C_3$$

$$C_4 = - \frac{\cos rl \sinh rl + \sin rl \cosh rl}{2(\cos rl . \cosh rl + 1)}$$

$$C_2 = - C_4$$

Substituting these constants into euqation 2.1.9 we can get the steady state response. This response represents the absolute displacement of any point of the cantilever beam under the action of the base displacement for all time, $t > 0$. The analytical values of maximum absolute displacement response for the cantilever structure considered in Figures 2 and 3 for various values of λ are shown in Table 2a as ω/p verses absolute maximum displacement/λ

2.2 Portal Frame - Single Storey :

2.2.0 Single Storey Portal Frame Subjected to Sinusoidal Base Displacement :

The portal frame shown in Figures 4 and 5 is idealised as a single degree of freedom system as shown in Figure 6 (8).

It is assumed that the joints of the frame are rigid. The vibration takes place in the plane of the structure only. The axial deformation and shear deformation are neglected. It is considered that the mass has no rotary inertia. The system is considered to be linear and conservative.

2.2.1 The Governing Equation of Motion :

The equation of motion for single degree of freedom system is given by

$$m\ddot{x} + kx = kx_g \qquad (2.2.4)$$

where m is the lumped mass.

$\ddot{x}$ is the acceleration

x is the absolute displacement of the mass

x_g is the base displacement

k is the equivalent stiffness of two columns of the portal which is equal to $24 \frac{EI}{l^3}$

where E is the Young's modulus of elasticity

I is the moment of inertia and

l is the length of the column.

2.2.2 The Free Vibration :

The equation which determines the natural frequency is the Equation 2.2.4 excluding the right hand side.

That is $\quad m\ddot{x} + lx = 0$

$$\ddot{x} + \frac{k}{m} x = 0$$

$$\ddot{x} + p^2 x += 0$$

where p is the natural frequency of the system equal to $\sqrt{k/m}$

The natural frequency p is given in the Graph 7.

2.2.3 Forced Vibration :

The equation to determine the forced vibration of the system is Equation (2.2.4) which could be rewritten as

$$\ddot{x} + p^2 x = p^2 \lambda \sin \omega t \quad \text{where } x_g = \lambda \sin \omega t \tag{2.2.5}$$

where λ is the amplitude of the forcing function. The solution of Equation (2.2.5) consists of two parts (1) homogeneous solution (2) the particular solution.

Therefore, the complete solution to Equation (2.2.5) is given by

$$x = A \sin pt + B \cos pt + p^2 \frac{\lambda \sin \omega t}{p^2 - \omega^2} \tag{2.2.6}$$

where A and B are constants of integration and they are determined from the initial condition.

Presuming that the structure has zero initial displacement and zero initial velocity, the initial conditions are $y(x,p) = 0$, $x(x,o) = 0$.

Substituting the intial conditions into Equation (2.2.5), we get

$$B = 0$$

$$A = -\frac{p^2 \lambda \omega}{(p^2-\omega^2)p}$$

$$= \frac{\lambda \omega}{p(1-\omega^2/p^2)}$$

Therefore the response

$$x = \frac{\lambda}{1-\omega^2/p^2}\left[\sin \omega t - \frac{\omega}{p} \sin pt\right] \tag{2.2.7}$$

The second term of the Equation (2.2.7) in the brackets is called the transient response which eventually dies out and the first term in the brackets is called the steady state response. The later is the subject of study here.

The maximum amplitude of the displacement response

$$\left|x_{max}\right| = \left|\max\left\{\frac{\lambda}{(1-\omega^2/p^2)} \sin \omega t\right\}\right|$$

$= \lambda/(1-\omega^2/p^2)$ since the maximum value of $\sin \omega t$ is $\pm$ 1.

Denoting $1/1-\omega^2/p^2$ as maximum dynamic load factor $(DLF)_{max}$, we get

$$|x_{max}| = \lambda.(DLF)_{max} \qquad (2.2.8)$$

The analytical values of ω/p versus $|x_{max}|/\lambda$ are tabulated in Table .3(a).

2.3 Portal Frame - Two Storey :

2.3.0 Portal Frame - Two Storey Subjected to Base Motion :

This frame is shown in Figure 7 and it is idealised as two degrees of freedom system with equivalent spring constant

$$k_1 = 24\frac{EI}{l^3} \text{ and } k_2 = 24\frac{EI}{l^3}$$

as shown in Figure 8.

Once again it is assumed that the system is linear and conservative and undamped. The joints are rigid and the axial, shear and rotational deformation are neglected.

2.3.1 The Governing Equation of Motion :

The governing equation of motion are

$$m_1\ddot{x}_1 + k_1 x_1 + k_2(x_1-x_2) = k_1\lambda\sin\omega t \qquad (2.2.9)$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0 \qquad (2.3.0)$$

where m_1 and m_2 are lumped masses as shown in Figure 8. $\ddot{x}_1$ and $\ddot{x}_2$ are accelerations, x_1 and x_2 are absolute displacements k_1 and k_2 are equivalent stiffness of the column of the portal frame and λ is the amplitude of the base displacement.

2.3.2 Free Vibration :

The free vibration study corresponds to the solution of homogeneous differ-ntial equations.

$$m_1\ddot{x}_1 + k_1x_1 + k_2 (x_1 - x_2) = 0$$
$$m_2\ddot{x}_2 + k_2(x_2-x_1) = 0$$

rearranging the above equations, we obtain

$$m_1\ddot{x}_1 + (k_1+k_2) x_1 - k_2x_2 = 0 \qquad (2.3.1)$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0 \qquad (2.3.2.)$$

Let $x_1 = \bar{x}_1 \sin pt$ and

$x_2 = \bar{x}_2 \sin pt$ be the solution of the equation (2.3.1) (2.3.2)

Therefore, the Equations (2.3.1) and (2.3.2.) becomes

$$=- p^2 m_1 \bar{x}_1 + (k_1+l_2) \bar{x}_1 - k_2\bar{x}_2 = 0$$

$$- p^2 m_2 \bar{x}_2 - k_2\bar{x}_1 + k_2\bar{x}_2 = 0$$

This set of equations can be written in the matrix form as

$$\begin{vmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{vmatrix} \begin{vmatrix} \bar{x}_1 \\ \bar{x}_2 \end{vmatrix} = p^2 \begin{vmatrix} m_1 & 0 \\ 0 & m_2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \qquad (2.3.3)$$

which is an eigen value problem. The above equation can be rewritten as

$$\begin{vmatrix} k_1 + k_2 - p^2 m_1 & -k_2 \\ -k_2 & k_2 - p^2 m_2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = 0 \qquad (2.3.4)$$

For non-trivial solution of equation (2.3.4) the determinant of square matrix of Equation (2.3.4) should be equal to zero.

i.e.

$$\begin{vmatrix} k_1 + k_2 - p^2 m_1 & -k_2 \\ -k_2 & k_2 - p^2 m_2 \end{vmatrix} = 0 \qquad (2.3.5)$$

Expansion of Equation (2.3.5) yields a quadratic equation in p^2 *and is called the characteristic equation. This is given* by

$$m_1 m_2 (p^4) - (k_1 m_2 + k_2 m_1 + k_2 m_2) p^2 + k_1 k_2 = 0$$

The solution to this quadratic equation yields the natural frequencies of the system. The natural frequency is given in Graph (10).

2.3.2 Forced Vibration :

The solution of the simultaneous differential equations (2.2.9) and (2.3.0) are determined as follows :

Let
$$x_1 = A \sin \omega t$$
$$x_2 = B \sin \omega t$$

Therefore, the Equations (2.2.9) and (2.3.0) become

$$(k_1 + k_2 - m_1\omega^2) A - k_2 B = k_1 \lambda$$

$$-k_2 A + (k_2 - m_2\omega^2) B = 0$$

From the above set of equations A, and B are given by

$$A = \frac{\alpha_1}{\beta}$$

$$B = \frac{\alpha_2}{\beta}$$

where
$$\alpha_1 = \begin{vmatrix} \lambda k_1 & -k_2 \\ 0 & k_2 - m_2\omega^2 \end{vmatrix}$$

$$\alpha_2 = \begin{vmatrix} k_1 + k_2 - m_1\omega^2 & k_1 \\ -k_2 & 0 \end{vmatrix}$$

$$\beta = \begin{vmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{vmatrix}$$

Therefore, the steady state part of the forced vibration response is given as

$$x_1 = \frac{\lambda k_1(k_2 - m_2\omega^2)}{m_1m_2\omega^4 - (k_1m_2 + k_2m_1 + k_2m_2)\omega^2 + k_1k_2} \quad \sin\omega t$$

and

$$x_2 = \frac{\lambda k_1k_2}{m_1m_2\omega^4 - (k_1m_2 + k_2m_1 + k_2m_2)\omega^2 + k_1k_2} \quad \sin\omega t$$

The maximum absolute value of x_1 and x_2 are given by

$$|x_1| = \frac{\lambda k_1(k_2 - m_2\omega^2)}{D} \tag{2.3.6}$$

$$|x_2| = \frac{\lambda k_1k_2}{D} \tag{2.3.7}$$

where $D = m_1m_2\omega^4 - (k_1m_2 + k_2m_1 + k_2m_2) + k_1 + k_1k_2$

Equations (2.3.6) and (2.3.7) are the response of the structure at mass points m_1 and m_2.

The analytical values of the ω/p versus the absolute displacement $|x_1|/\lambda$ and $|x_2|/\lambda$ are given in Table (4a).

CHAPTER 3
EXPERIMENTAL MODEL
EXPERIMENTAL MODEL

3.1.0 Cantilever beam subjected to sinusodal base motion:

The contilever beam considered is shown in fig. 1, 2 and 3. Figure 1 shows the beam with the fixure which has been fabricated to mount the beam on the Shaker. Figures 2 and 3 respectively show the dimensions of the beam considered made of steel and aluminium. The full size model has been fabricated .

3.1.1 The cantilever with the fixure is mounted on to the shaker table. The motion of the table is effected by operating the shaker control. The displacement and the frequency are read from the meters provided on the control panel. However, for the purpose of cheking the correctness a M.B. vibration meter with a M.B. vibration pick up are used. The pick up is mounted on the table and the signal is fed into the vibration meter which records the displacement. To check the frequency shown by the control meter an electronic- strobotac is used.

For constant table displacement the frequency is varied and the system displacement at the free end of the cantilever is measured through a travelling microscope. The displacement obtained experimentally are recorded in table 2b, 2c, 2d, for aluminium and table 2f, 2g, 2h for steel for the maximum base displacement of .01 inch, .02 inch

d.03 inc. The first two mode shapes are shown in photo numbers 1 and 2.

3.1.2 The sequence of operations for the measurement of system displacement are as follows:

1. The cantilever beam with the fixure is mounted on the shaker table. The telescope of the travelling microscope is focused to a point marked at the tip of the beam, and the table motion is given to the fixed end of the beam.

2. While the structure is in vibration strobotac light is passed on the structure and the frequency of the light is adjusted until the frequency of the beam coincides with the frequency of the light. This is ensured when the structure appears to be stationary when viewed through the telescope.

3. To measure the displacement the frequency of the light is lightly reduced such that the structure takes a slow motion. Then the telescope is lowered down or raised up such that the cross hair coincides with the extreme position occupied by the point on the structure. The amount of movement by the telescope is measured on the verticle scale.

3.2.0 Portal frame: Single story subjected to base displacement.

The single story portal frame as shown in figure 4 and 5 is fabricated out of steel pflats in full size. The frame is attached to a rigid base. A mass is added to the horizontal flat and the corners of the frame are braced so as to provide rigidity to the joints. This also restraints the mass from rotation.

3.2.1 To study the dynamic response of this portal frame, M.B. vibration exciter is used. The frame is fixed on to the table which is turn is subjected to sinusoidal displacement.

3.2.2 The same sequence of operations as that for the cantilever beam is adopted to measure the displacement and frequency of the portal frame using the travelling microscope and the strobotac. The measured displacements are recorded in Tables 3b, 3c and 3d for the maximum table displacements of 0.04 inch, 0.06 inch and 0.08 inch. The mode shape is shown in photo number 3.

3.3.0 Portal frame -- Two story subjected to base displacement.

The two storied portal frame is also made of steel flats and fabricated in full size as shown in figure 7.

Two masses m_1 and m_2 are attached and braced at l_1 and $l_1 + l_2$ from the base.

3.3.1 The frequency and the displacements are measured following the same sequence of operations as that of cantilever beam and portal frame except that the displacement is measured at two mass points, one at l_1 from the base and the other $l_1 + l_2$ from the base.

The experimental results are given in the tables 4b and 4c for base displacement of 0.04 inch, 0.06 inch and 0.08 inch.

The first mode shape of the frame is shown in photo number 4.

CHAPTER 4

EXPERIMENTAL SET UP

The following instruments are used in the present work. This is shown in photos 5 and 6.

1. Unholds Dickie vibration system
2. M.B. vibration exciter.
3. Travelling Microscope
4. Strobotac

The brief description of these experement are given in this chapter.

4.1.0 Unholds Dickie Vibration System

4.1.1 System specification.:

Generated force--- continuous rating = 180 lb peak

Armature assembly effective weight = 4.3 lb

Maximum free table acceleration = 42 g.

Shaker stroke --- snubber to snubber = 1 inch

Free table axial resonance frequency = 5000 cps.

This vibrating system consists of three units; one of them is called master console and the other two are called slave consoles. Each console consists of a shaker table. All the three consoles or any one console could be put into operation.

The master console consists of a shaker control, an electronic power amplifier and a Dial - A - Gain

1.2 The Shaker Table

4.1.2 The Shaker Table: The Shaker table is the heart of the system. The figure 9 shows the cross sectional elevation of the shaker table.

The field coils are connected to a d.c. supply. When an alternating current passes through the driver coil, an electromagnetic force is generated in the driver coil wires which causes the table to move up and down. The frequency of the alternating current determines the frequency of the vibration in the table.

The shaker is trunion mounted so that the shaker could be oriented either vertically of horizontally or in any other position between these two limits. An accelerometer is attached to the internal end of the table which in conjunction with the Dial - A- Gain and the automatic shaker control, vibration meter indicates the shaker table vibrating displacement or velocity or acceleration.

4.1.3 Electronic Power Amplifier : This is provided to generate the required force by amplifying the low power, low voltage generated by the oscillator. This consists of multi-amplifier section and therefore, the system can be operated with a single amplifier section or a multi-amplifier section. The amplifier is rated at 500 watts output and operated with 580 volts B^+ on the output tubes.

4.1.4 Shaker Control : This consists of an oscillator which originates the signal voltage that is fed to the power amplifier which in turn causes the alternating current to flow through the driver coil. The driver coil current

causes the table motion. Therefore the frequency and the magnitude of the table motion is determined by the frequency and the magnitude of the a.c. current.

4.1.5 Dial -A- Gain: This is used to monitor the builtin accelerometer or any other accelerometer mounted on the table. This helps in fine adjustment of the table motion.

4.2.0 M.B. Vibration Exciter: The construction and the principle of M.B. vibration exciter is same as that of Unholds- Dickie vibration system except that the rated force generated is only 25 lb peak. The free table motion is 0.5 inch. Also it is provided with a permanent magnet to eliminate the need for power source to keep an electromagnet energised.

4.3.0 Travelling Microscope: The travelling microscope consists of a telescope which slides over a verticle scale graduated in centimeter with a vernier of least count 0.001 centimeter. Together with the vertical scale the telescope slides over a horizontal scale graduated in centimeter which also carries a vernier which could read a least count of 0.001 cm. . With the help of magnifying lenses the scale readings can be read to an accuracy of 0.001 centimeter.

4.4.0 Strobotac : This is an electronic device that generates the repetative light flashes of extremely short duration. The rate of flash is adjustable permitting the observation of any repetative or periodic motion as if it were stopped or moving very slowly.

The power supply required is 105 - 125 volts. The frequency range that could be covered is from 110 revolution per minute to 25000 revolution per minute.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1.0 Cantilever Beam Subjected to Sinusoidal Ground Motion :

The theoretical and the experimental values of the ratio of the maximum absolute displacement of the cantilever to the ground displacement are given in Tables 2a, 2b, 2c, 2d, 2f, 2g and 2h for aluminum and steel cantilevers.

The ratio of the absolute displacement to the maximum ground displacement is plotted against the ratio of the forcing frequency to the fundamental natural frequency and these plots are shown in Graphs no. 1 to 6.

The plots reveal the following :

(1) The observed maximum displacement of the cantilever occurs at a value ω/p which is less than the theoretical value of $\omega/p = 1.0$. This is due to the fact that damping has been neglected in the present work while considering the theoretical values. However, in the experimental observation inherent damping is present.

In order to check whether the shift of ω/p point corresponding to the maximum displacement to the left of the theoretical value is due to the above surmise, the damping coefficient is worked out and shown on Graph 1. Using this

value of the damping coefficient damped natural frequency is calculated which is lower than the theoretical (undamped) natural frequency which corresponds to the numerical value of the shift observed.

(2) The theoretical and observed values of maximum displacement differ by 6 to 10 percent both in the case of aluminium and steel cantilevers. However, it is observed that for $\omega << p$ the theoretical values for aluminium cantilever are lower than those obtained experimentally. This could be attributed to the support condition because in practice perfect fixity is difficult to attain. Furthermore, for $\omega >> p$ the theoretical values are higher than those obtained experimentally. This shows that when the structure vibrates at frequency lower than the frequency of the forcing function, the damping plays a predominant role.

In the case of steel cantilever the observed values of maximum displacement are throughout higher than the theoretical values. This is due to the fact that the damping of steel plays a predominant role throughout.

5.2.0 Simple Storey Portal Frame Subjected to Ground Motion :

The experimental and the theoretical values of the ratio of the maximum absolute displacements are given in Tables 3a, 3b, 3c and 3d.

Similar plots are made in the case of cantilever beams and these plots are shown in Graphs 7, 8 and 9.

Similar to the observations made in the case of cantilever beams, it is seen here too that the maximum displacement of the structure occurs at a frequency of 29 cps as against the exoected theoretical frequency of 29.85 cps. This shift, once again, is due to the inherent damping in the structure.

The observed values of the displacement are all through lower than the theoretical values. This too is because of the inherent damping in the structure.

Obviously putting of a rigid strut at the base which in turn is connected to the rigid fiure sufficiently ensured the considered boundary conditions as fixed.

5.3.0 Two Storey Portal Frame :

The theoretical and experimental values of maximum absolute displacements are given in Tables 4a, 4b, and 4c.

The maximum absolute displacement are plotted against the frequency of the system and these plots are shown in Graphs 10, 11 and 12.

The observations are made only up to 45 cycles per second due to the experimental difficulties.

The plots reveal that the maximum displacement occur at a frequency 3 cycles less than the fundamental natural frequency obtained by theory. Also, the observed values of the maximum displacements are well wit in allowable limit of error. The shift of the frequency is attributed to the inherent damping present in the structure. Here, too, the assumed fixed support condition is sufficiently ensured by putting of a rigid base to the frame and the rigid fixure.

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10. D.E.Hudson ' Dynamic Tests of full scale models' A report of research carried out by the earthquake engineering research institute under State of California Standard agreement number 2163 for the State of California Department of Public Works, Division of Architecture, State Architect- Sept. 1961

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4. Den Hartog 'Mechanical Vibrations'
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John Wiley Publications -

TABLE 1a

ALUMINIUM

rl	Natural frequency cycles per second	Mode shapes defined by			
		A1	A2	A3	A4
1.785	64.4	1.00	-0.74	-1.00	0.74
4.694	393.4	1.00	-1,02	-1.00	1.02

TABLE 1b

STEEL

1.785	22.44	1.00	-0.74	-1.00	0.74
4.694	140.6	1.00	-1,02	-1.00	1.02

FORCING FREQUENCY / NATURAL FREQUENCY AGAINST ABSOLUTE DISPLACEMENT
GROUND DISPLACEMENT

TABLE 2a

MATERIAL : ALUMINIUM
THEORITICAL VALUES

Forcing Frequency/ Natural Frequency	Maximum absolute displacement/ Ground displacement
0.161	1.039
0.322	1.177
0.482	1.448
0.643	2.060
0.804	3.721
0.816	4.020
0.832	4.420
0.850	4.880
0.865	5.440
0.881	6.130
0.896	7.060
0.912	8.340
0.929	10.160
0.945	12.960
0.961	17.810
0.976	28.280
0.994	67.240
1.010	180.130
1.027	40.660
1.042	23.740
1.059	17.140
1.072	13.670
1.125	8.840
1.286	5.100
1.4472	4.050
1.608	3.520

MATERIAL : ALUMINIUM

EXPERIMENTAL VALUES

TABLE 2b

$\lambda = 0.01$ INCH

Frequency / Natural frequency	Displacement/ (absolute) Ground displacement
0.1811	1.1
0.337	1.26
0.401	1.34
0.478	1.535
0.55	1.69
0.642	2.16
0.732	2.95
0.809	4.25
0.89	7.28
0.977	22.8
1.05	18.9
1.05	18.9
1.19	6.06
1.294	4.72
1.357	3.58

TABLE 2c

$\lambda = 0.02$ INCH

Frequency / Natural frequency	Displacement/ (absolute) Ground displacement
0.174	1.1
0.337	1.26
0.405	1.359
0.491	1.572
0.55	1.79
0.645	2.185
0.73	2.78
0.805	4.14
0.897	7.29
0.977	22.3
1.11	8.5
1.222	5.6
1.303	4.82
1.368	3.6

TABLE 2d

$\lambda = 0.03$ INCH

Frequency/ Natural frequency	Displacement / (absolute) Ground displacement	Frequency/ Natural frequency	Displacement / (absolute) Ground displacement
0.176	1.142	0.817	4.21
0.337	1.271	0.898	8.3
0.396	1.38	0.977	22.5
0.496	1.56	1.128	8.3
0.583	1.88	1.208	5.51
0.642	2.16	1.327	4.47
0.748	3.16	1.384	3.96

MATERIAL : STEEL

EXPERIMENTAL VALUES

TABLE 2f

Ground Displacement =0.01 inch

Frequency / Natural frequency	Displacement/ (absolute) Ground displacement
0.341	1.26
0.434	1.378
0.55	1.751
0.62	2.740
0.76	3.743
0.868	4.92
0.900	5.798
0.975	13.254
0.984	18.75
1.032	6.35
1.26	5.12
1.515	3.66
1.755	3.11
1.995	2.99
2.18	2.96
2.315	2.795
2.57	2.795
2.89	2.795

TABLE 2g

Ground displacement = 0.02 inch

Frequency / Natural frequency	Displacement / (absolute) Ground displacement
0.33	1.33
0.400	1.40
0.455	1.435
0.481	1.746
0.550	2.119
0.643	2.252
0.731	3.214
0.799	4.263
0.898	7.362
0.91	9.45
0.95	15.0
0.96	22.49
1.02	5.618
1.05	13.78
1.11	8.51
1.19	5.6
1.46	3.92
1.69	3.23
2.09	2.93
2.54	2.78
2.75	2.78
2.93	2.78

TABLE 2b

Ground displacement =0.03 inch

Frequency / Natural frequency	Displacement / (absolute) Ground displacement
0.35	
0.335	1.25
0.38	1.27
0.455	1.56
0.589	2.10
0.641	2.248
0.743	2.325
0.800	4.25
0.890	8.48
0.97	22.5
1.03	10.21
1.10	8.3
1.0	5.245
1.275	4.96
1.30	4.51
1.40	4.02
1.43	3.82
1.68	3.3
2.11	2.9
2.53	2.78
2.75	2.78
2.91	2.78

SINGLE STOREY

Natural Frequency = 29.85 cycles /sec.

TABLE 3a

THEORETICAL VALUES

Frequency / Natural frequency	Displacement / (absolute) Ground displacement
0.301	1.125
0.334	1.14
0.401	1.24
0.469	1.31
0.535	1.41
0.602	1.58
0.667	1.84
0.735	2.24
0.803	2.88
0.869	4.20
0.936	8.14
0.97	16.19
1.002	16.9
1.037	12.51
1.07	7.31
1.14	3.45
1.205	2.22
1.271	1.631
1.475	0.875

TABLE 3b

EXPERIMENTAL VALUES

Ground displacement = 0.04 inch

Frequency / Natural frequency	Displacement / (absolute) Ground displacement
0.323	1.08
0.378	1.12
0.445	1.21
0.501	1.25
0.556	1.30
0.611	1.45
0.67	1.65
0.731	2.01
0.790	2.51
0.862	3.36
0.924	4.40
0.973	6.78
1.002	5.61
1.045	5.02
1.121	3.41
1.205	2.01
1.299	1.35
1.47	0.81

TABLE 3c

EXPERIMENTAL VALUES

Ground displacement=0.06 inch

Frequency / Natural frequency	Displacement/ (absolute) Ground displacement
0.323	1.08
0.378	1.12
0.445	1.20
0.501	1.23
0.556	1.28
0.611	1.41
0.67	1.61
0.67	1.61
0.731	1.98
0.790	2.47
0.862	3.33
0.924	4.31
1.045	4.95
1.121	3.38
1.205	2.01
1.299	1.34
1.47	0.78

TABLE 3d

EXPERIMENTAL VALUES

Ground displacement = 0.08 inch

Frequency / Natural Frequency	Displacement / (absolute) Ground displacement
0.323	1.07
0.378	1.11
0.445	1.21
0.501	1.24
0.556	1.27
0.611	1.42
0.67	1.60
0.67	1.60
0.731	1.96
0.790	2.48
0.862	3.31
1.045	4.93
1.121	3.36
1.295	2.00
1.299	1.33
1.47	0.79

TWO DEGREE OF FREEDOM OF SYSTEM

TABLE 4a

THEORATICAL VALUES

Frequency cycles / second	DISPLACEMENT (absolute) λ = 0.1 cm. Mass A	Mass B	λ = 0.15 cm. Mass A	Mass B
8	0.109	0.108	0.164	0.161
20	0.177	0.156	0.265	0.233
30	0.893	0.651	0.134	0.977
31	1.613	1.147	2.720	1.720
31.5	2.715	1.905	4.073	2.658
32	8.644	5.982	12.966	8.973
32.5	-7.263	-4.956	-10.895	-7.434
33	-2.553	-1.717	-3.829	-2.576
34	-1.111	-0.724	-1.665	-1.086
40	-0.255	-0.132	-0.382	-0.198
48	-0.134	-0.041	-0.201	-0.062
60	-0.101	-0.008	-0.151	0.012
72	-0.171	0.095	-0.256	0.143
76	-0.428	0.316	-0.642	0.473
77	-0.801	0.628	-1.202	0.941
77.5	-1.490	1.202	-2.224	1.803
78	-14.005	11.622	-21.009	17.433
78.5	1.815	1.549	2.722	-2.323
79	0.936	-0.733	1.253	-1.099
80	0.391	-0.361	0.586	-0.542
86	0.058	-0.077	0.087	-0.012
100	0.018	-0.037	0.028	-0.055

TABLE 4a (Contd.)

Frequency cycles / second	Displacement (absolute)	
	λ = 0.2 cm.	
	Mass A	Mass B
8	0.219	0.215
20	0.354	0.311
30	1.786	1.302
31	3.226	2.294
31.5	5.431	3.810
32	17.288	11.964
32.5	-14.527	-9.912
33	-5.107	-3.434
34	-2.220	-1.448
40	-0.510	-0.265
48	-0.268	-0.082
60	-0.201	0.166
72	-0.341	0.191
76	-0.856	0.631
77	-1.602	1.255
77.5	-2.981	2.404
78	-28.011	23.244
78.5	3.630	-3.077
79	1.671	-1.466
80	0.782	-0.723
88	0.116	-0.154
100	0.037	-0.074

TABLE 4b

EXPERIMENTAL VALUES FOR MASS A

FREQUENCY		DISPLACEMENT (absolute)		
RPM	cycles / second	λ =0.1 cm.	λ =0.15 cm	λ =0.2 cm.
410	6.83	0.115	0.177	0.221
610	10.17	0.125	0.187	0.238
930	15.5	0.15	0.221	0.296
1160	19.33	0.17	0.254	0.336
1400	23.33	0.221	0.333	0.436
1570	26.17	0.339	0.501	-
1620	27.0	0.491	-	-
1710	28.5	0.537	-	-
1750	29.17	0.366	0.521	-
1800	30.0	0.275	0.391	0.541
2040	34.0	0.172	0.248	0.318
2340	39.0	0.123	0.186	0.241
2660	44.33	0.115	0.174	0.231

TABLE 4c

EXPERIMENTAL VALUES FOR MASS B

410	6.83	0.105	0.151	0.213
610	10.17	0.110	0.164	0.222
930	15.5	0.117	0.175	0.236
1160	19.33	0.141	0.208	0.278
1400	23.33	0.185	0.275	0.365
1570	26.17	0.271	0.402	0.536
1620	27.00	0.310	0.456	-
1710	28.5	0.395	-	-
1750	29.17	0.331	0.491	-
1800	30.0	0.260	0.380	0.510
2040	34.0	0.151	0.223	0.285
2340	39.0	0.084	0.125	0.165
2660	44.33	0.024	0.035	0.046

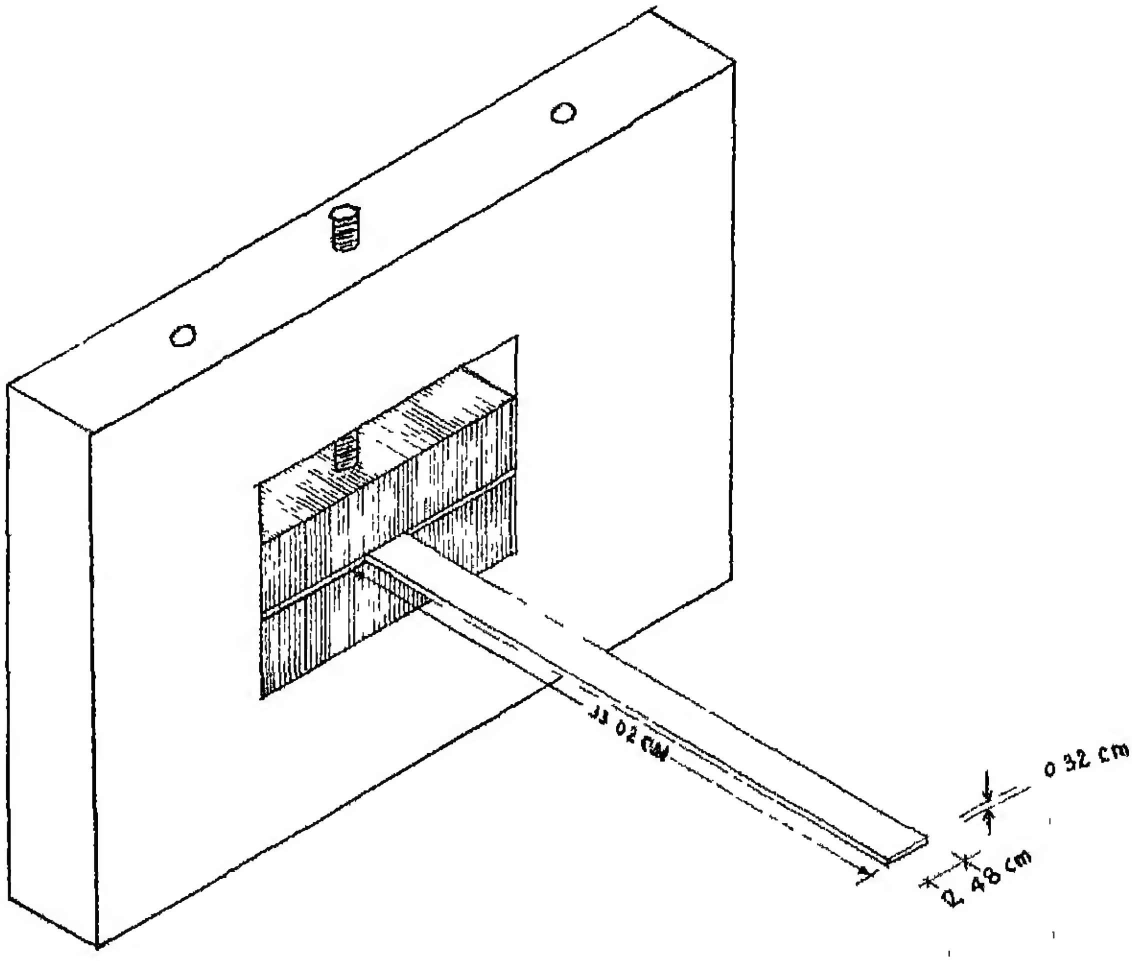


FIGURE. 1

STEEL

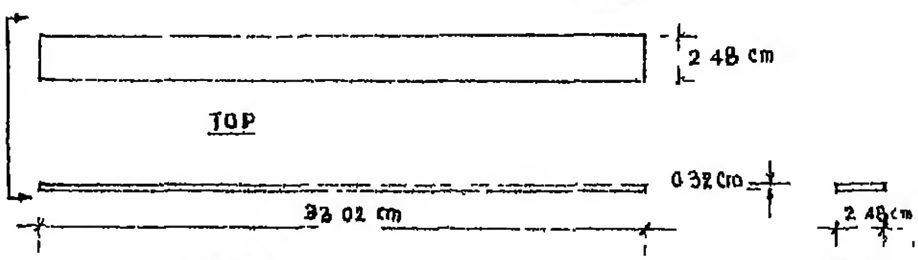


FIGURE. 2

ALUMINIUM.

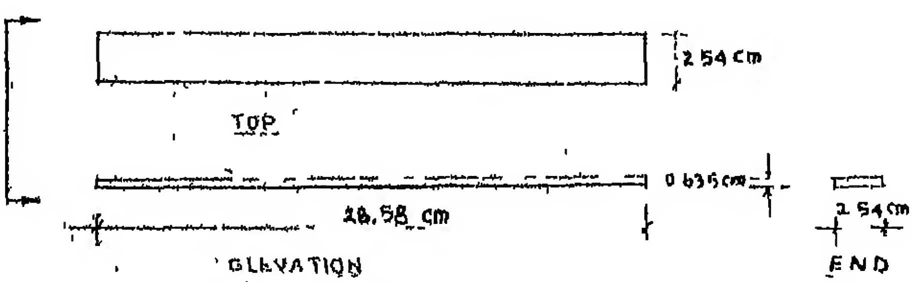


FIGURE. 3.

PORTAL FRAME   ONE STOREY

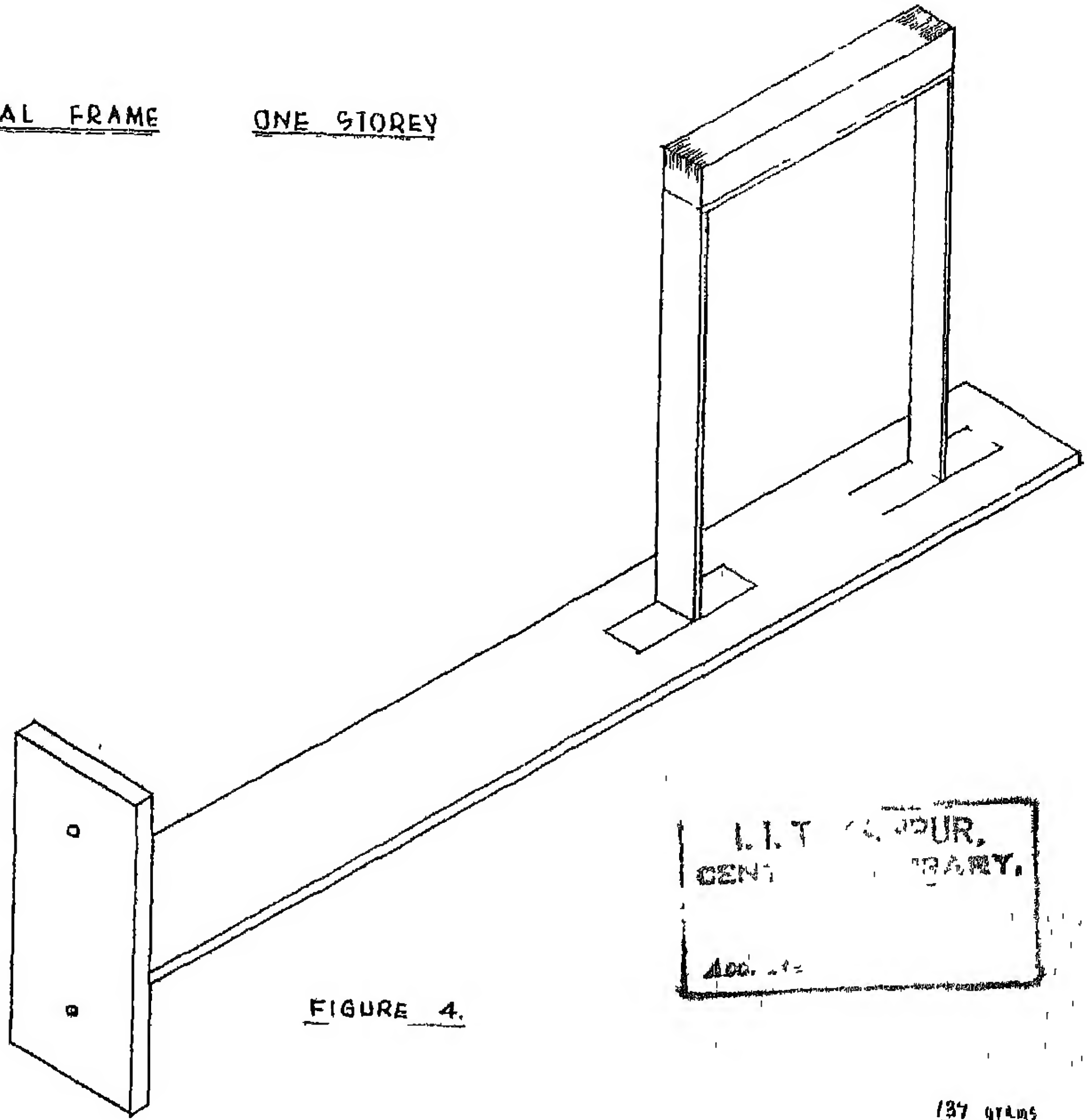
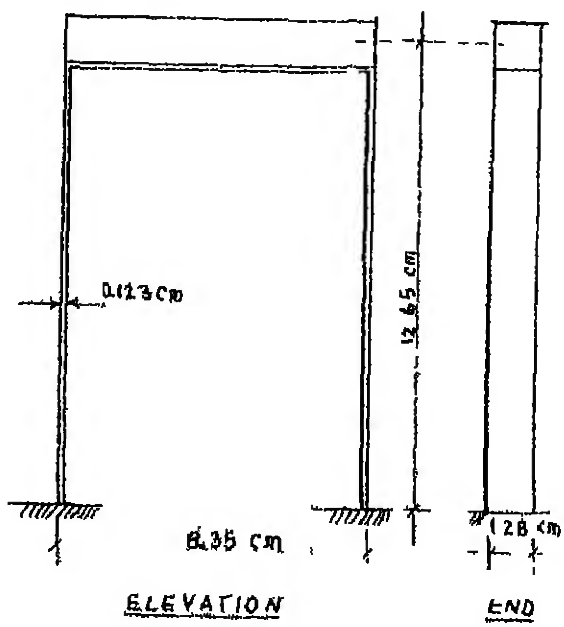


FIGURE 4.

FIGURE 5



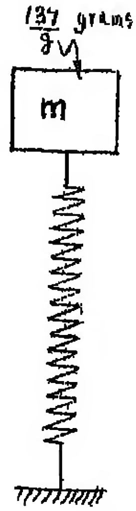


FIGURE. 6.

PORTAL FRAME TWO STOREYS

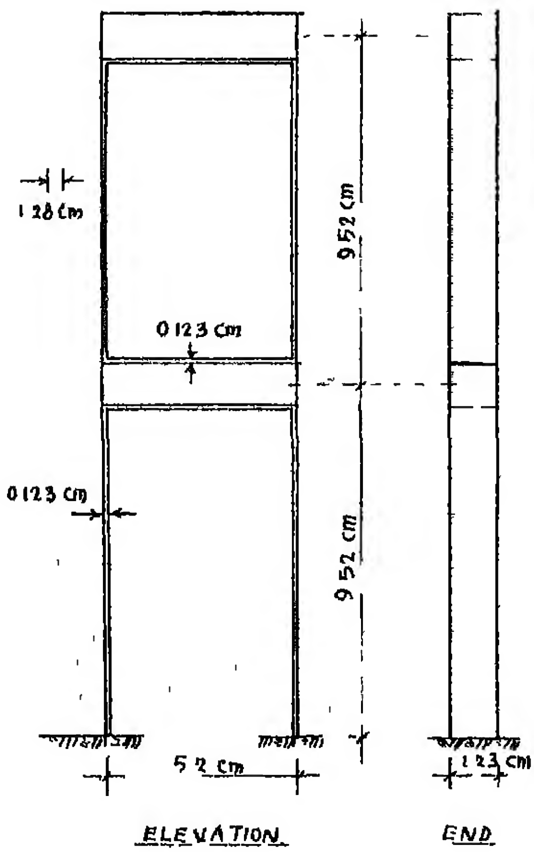


FIGURE 7

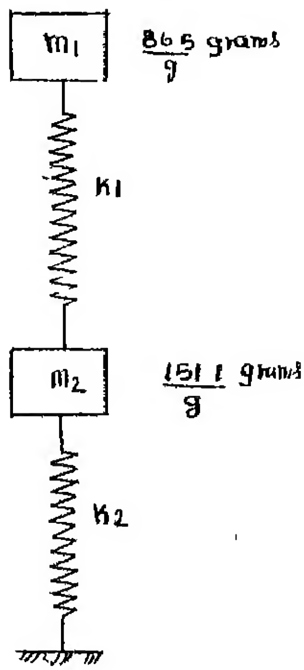


FIGURE 8

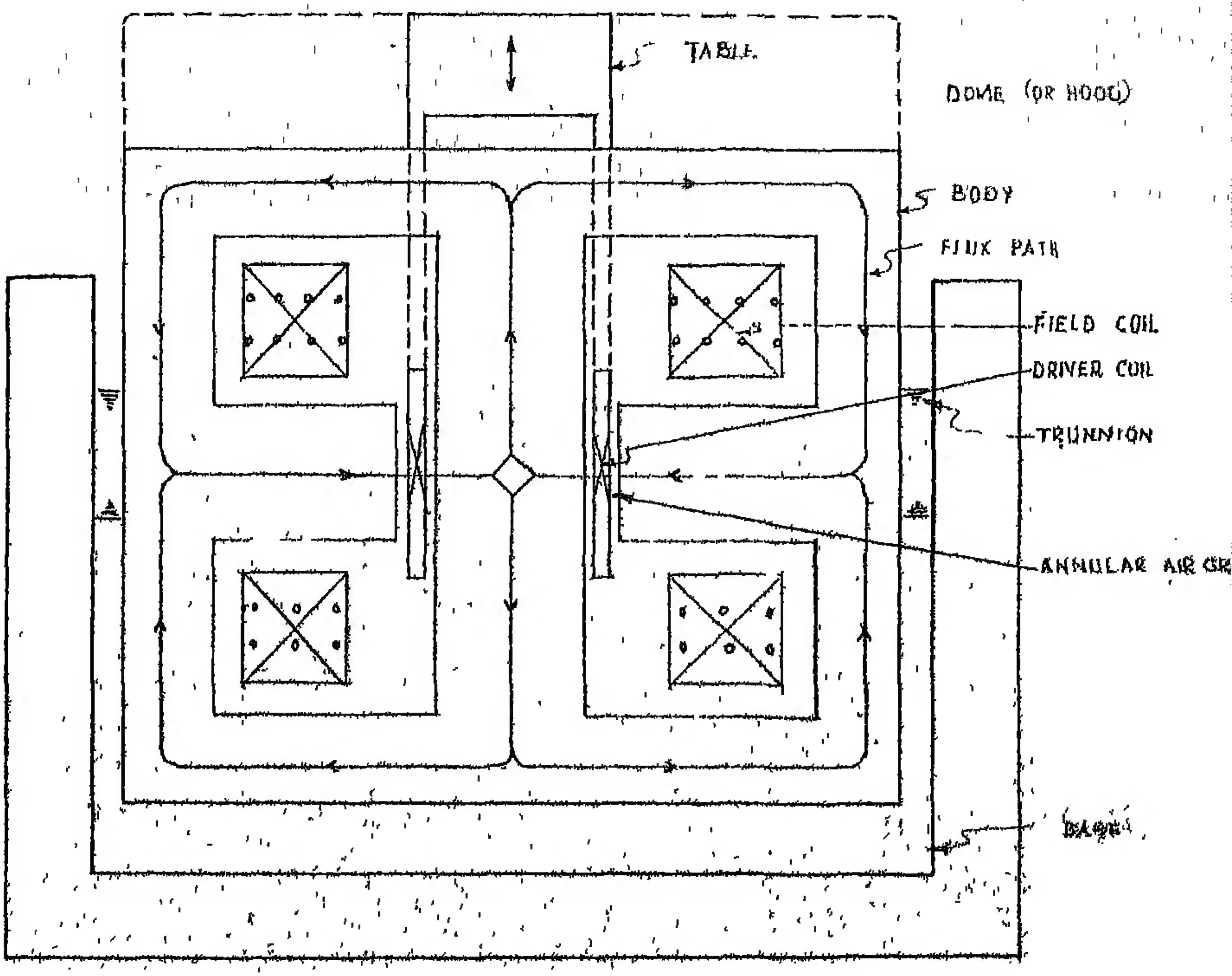


FIGURE 9

SHAKER SCHEMATIC DIAGRAM

CANTILEVER

GRAPH. 1

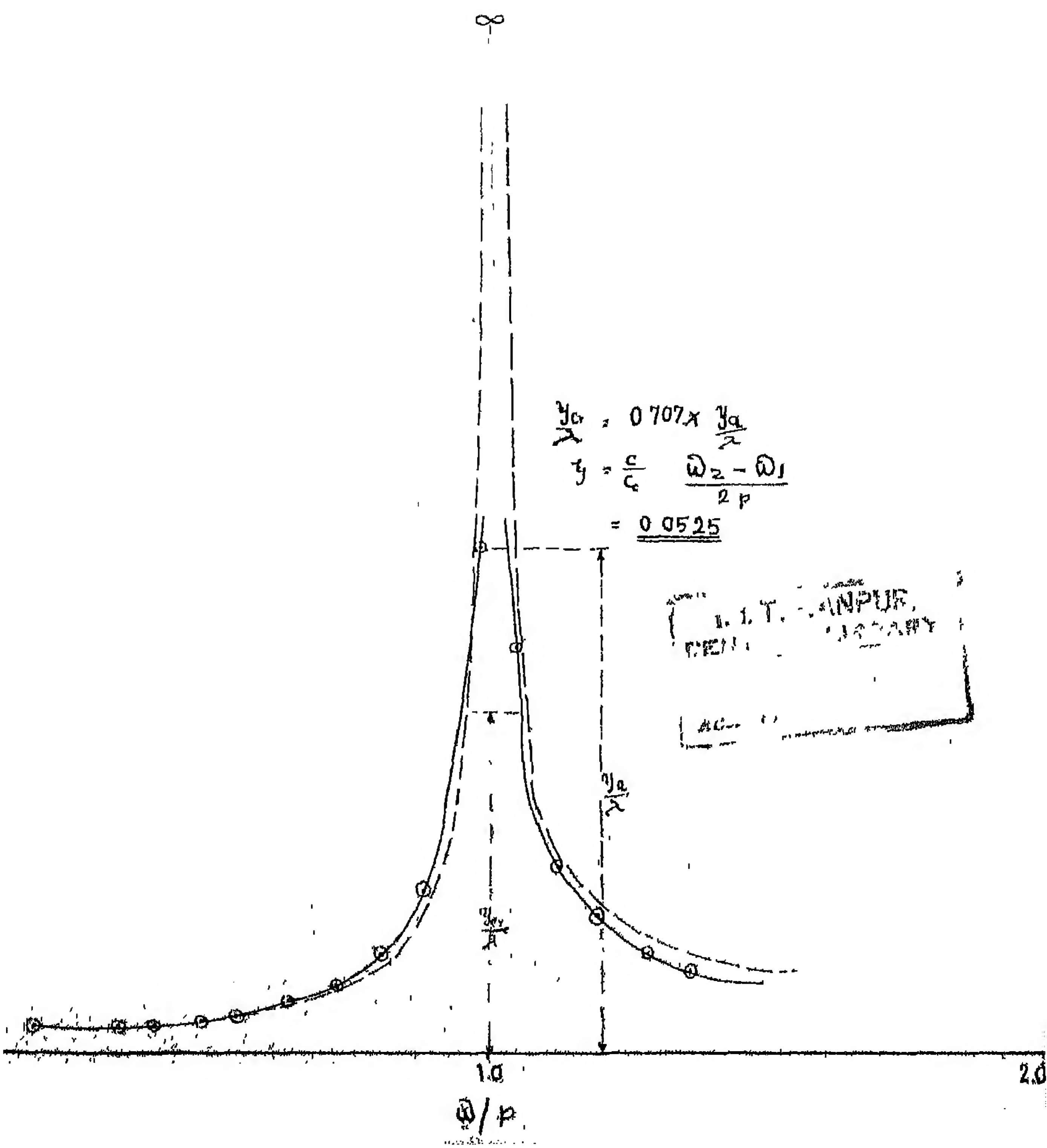
MATERIAL ALUMINIUM

PLOT OF RATIO OF THE FORCING FREQUENCY TO THE NATURAL FREQU AGAINST RATIO OF THE ABSOLUTE DISPLACEMENT TO THE GROUND DISPLACEMENT

FOR λ = 0.01 inch

------ Theoratical

- - - - - Experimental



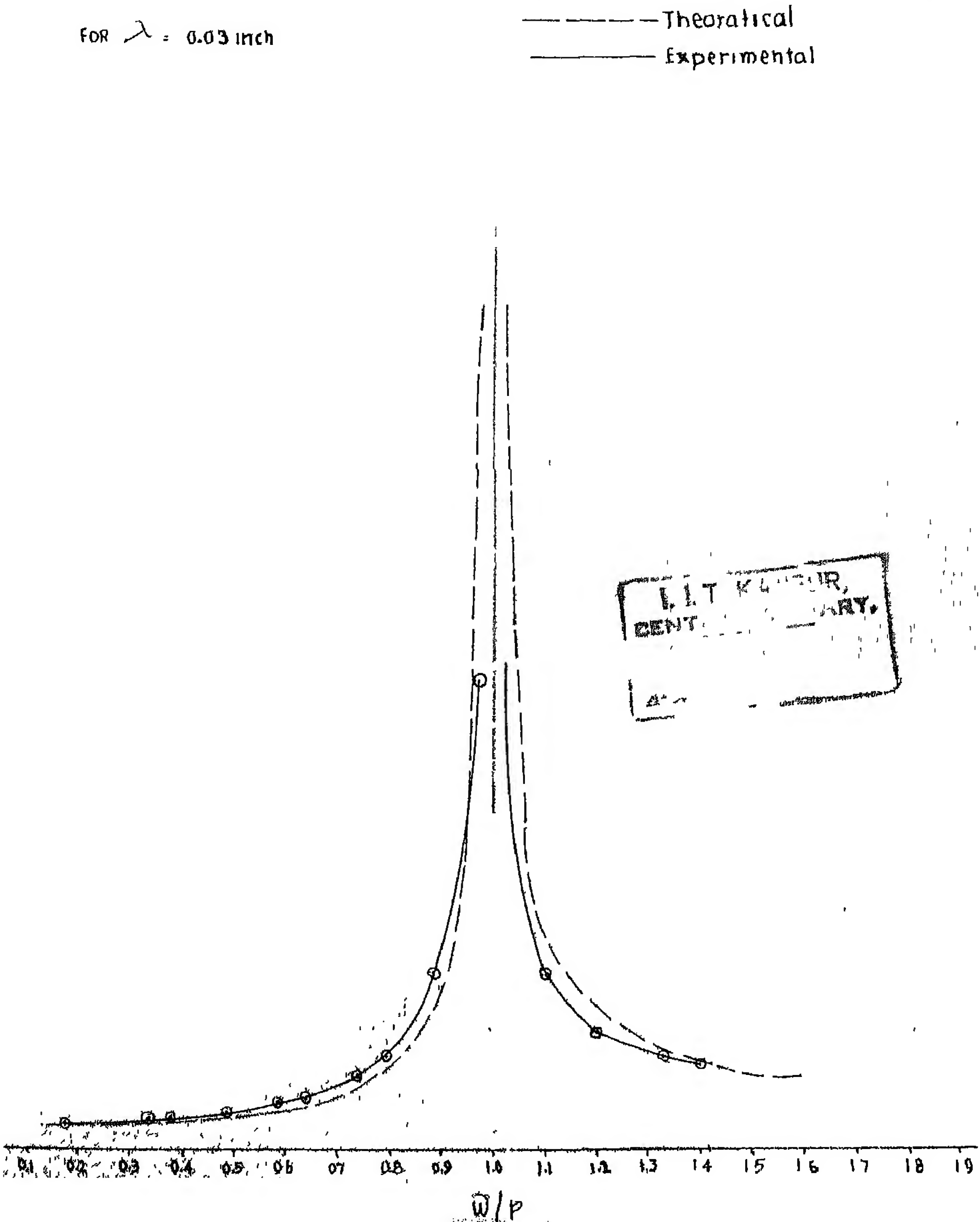
GRAPH - 3

CANTILEVER

MATERIAL ALUMINIUM

PLOT OF RATIO OF THE FORCING FREQUENCY TO THE NATURAL FREQUENCY AGAINST THE RATIO OF THE ABSOLUTE DISPLACEMENT TO GROUND DISPLACEMENT

FOR λ = 0.03 inch



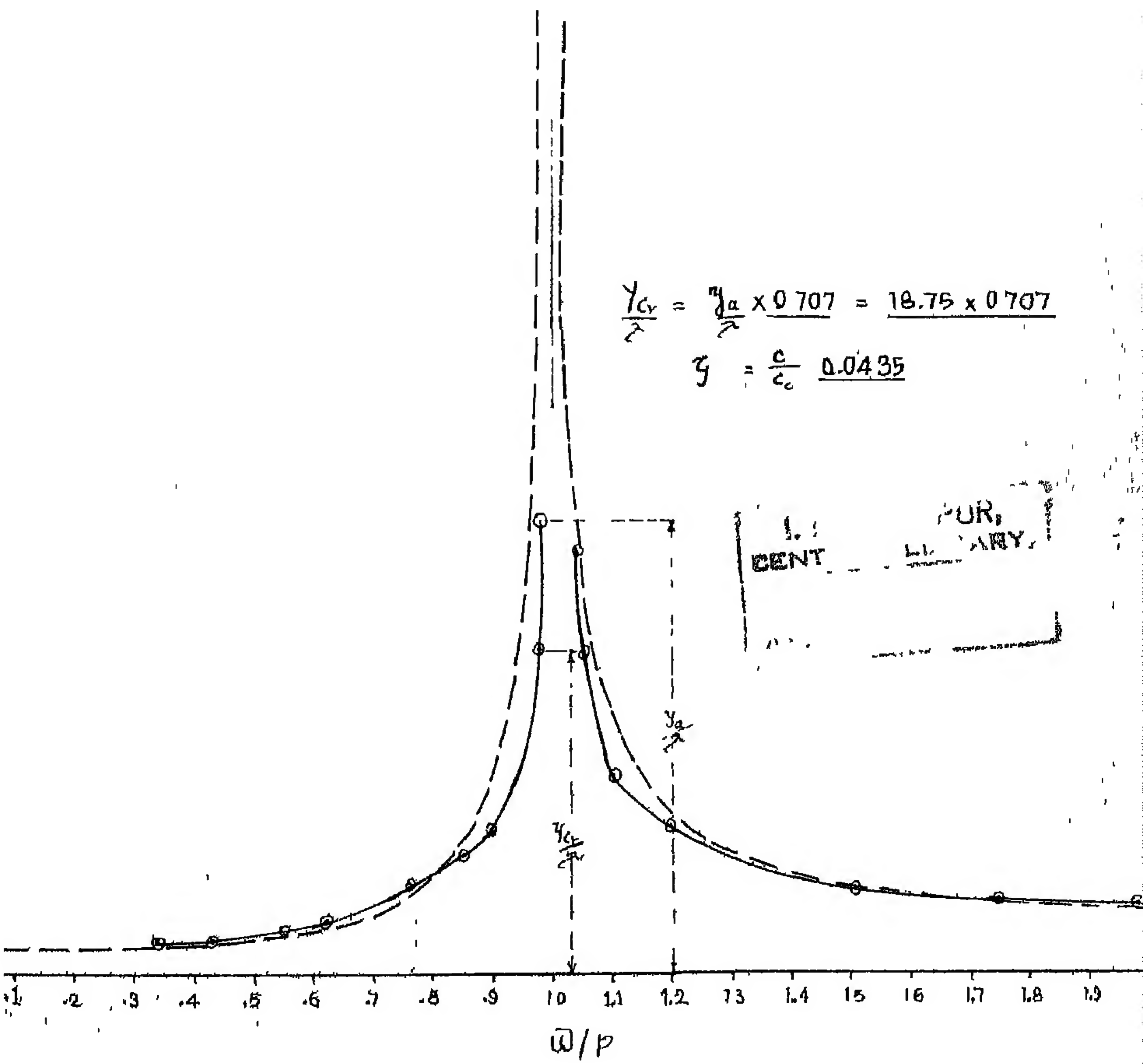
MATERIAL STEEL **GRAPH-4**

PLOT OF THE RATIO OF THE FORCING FREQUENCY TO THE NATURAL FREQUENCY AGAINST THE RATIO OF THE DISPLACEMENT TO THE GROUND DISPLACEMENT

FOR λ : 0.01 inch

- - - - - - THEORATICAL

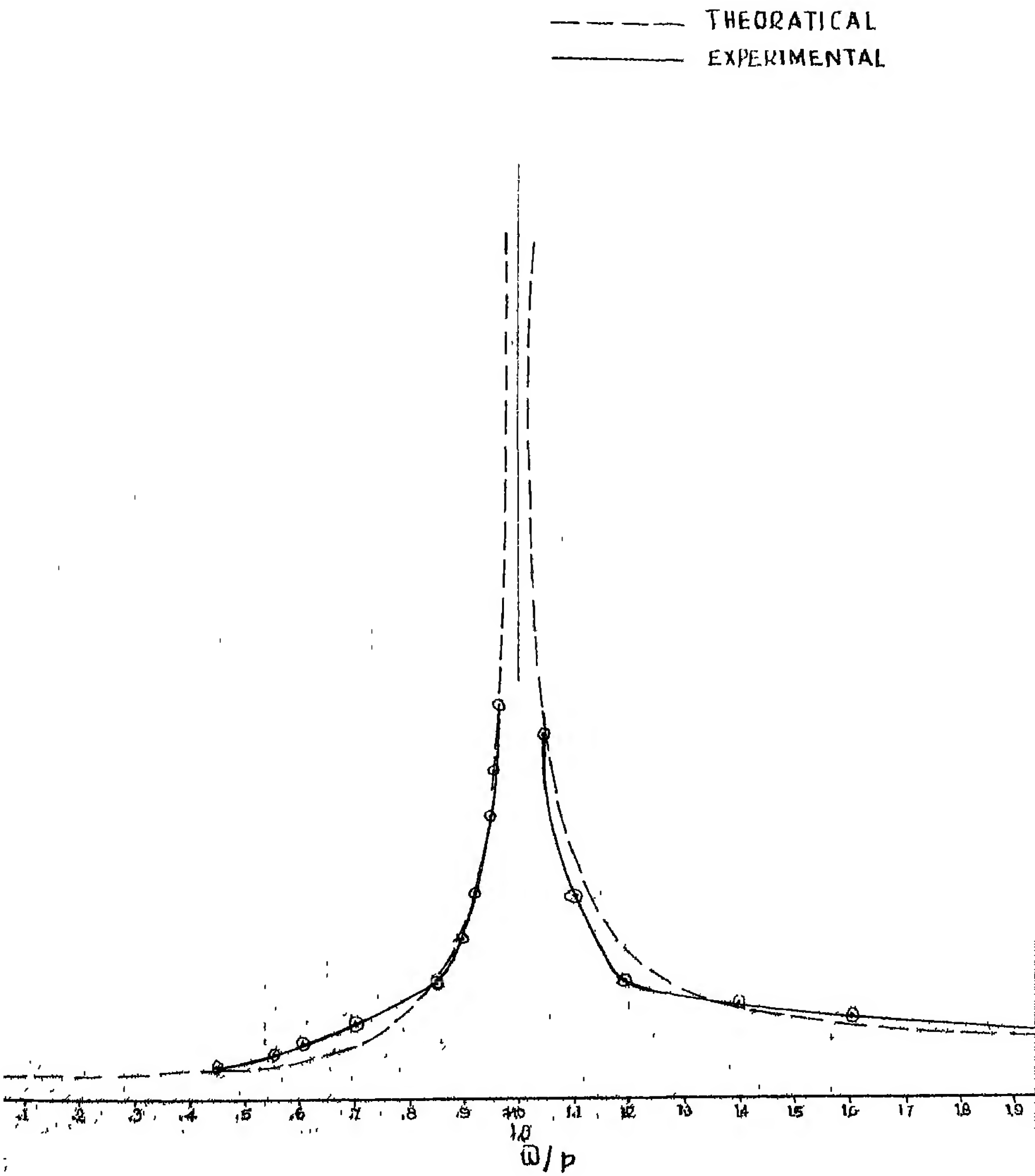
———— – EXPERIMENTAL



MATERIAL STEEL GRAPH·5

PLOT OF THE RATIO OF THE FORCING FREQUENCY TO THE NATURAL FREQUEN
AGAINST THE RATIO OF THE DISPLACEMENT TO THE GROUND DISPLACEMENT

FOR λ - 0 02 inch



GRAPH-6

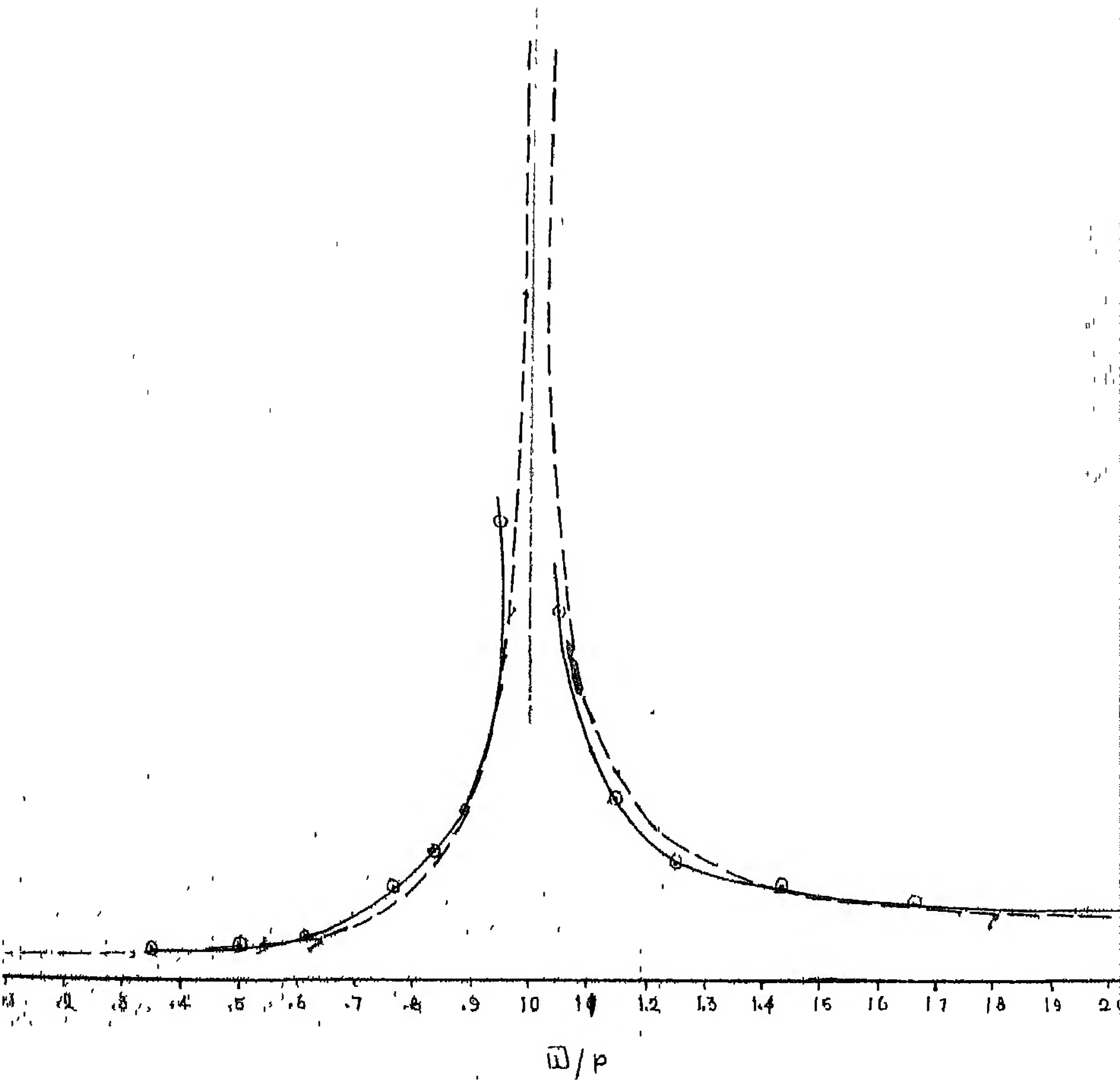
MATERIAL STEEL

PLOT OF THE RATIO OF THE FORCING FREQUENCY TO THE NATURAL FREQUENCY AGAINST THE RATIO OF THE DISPLACEMENT TO THE GROUND DISPLACEMENT

FOR λ . 003 inch

– – – – THEORATICAL

——— EXPERIMENTAL

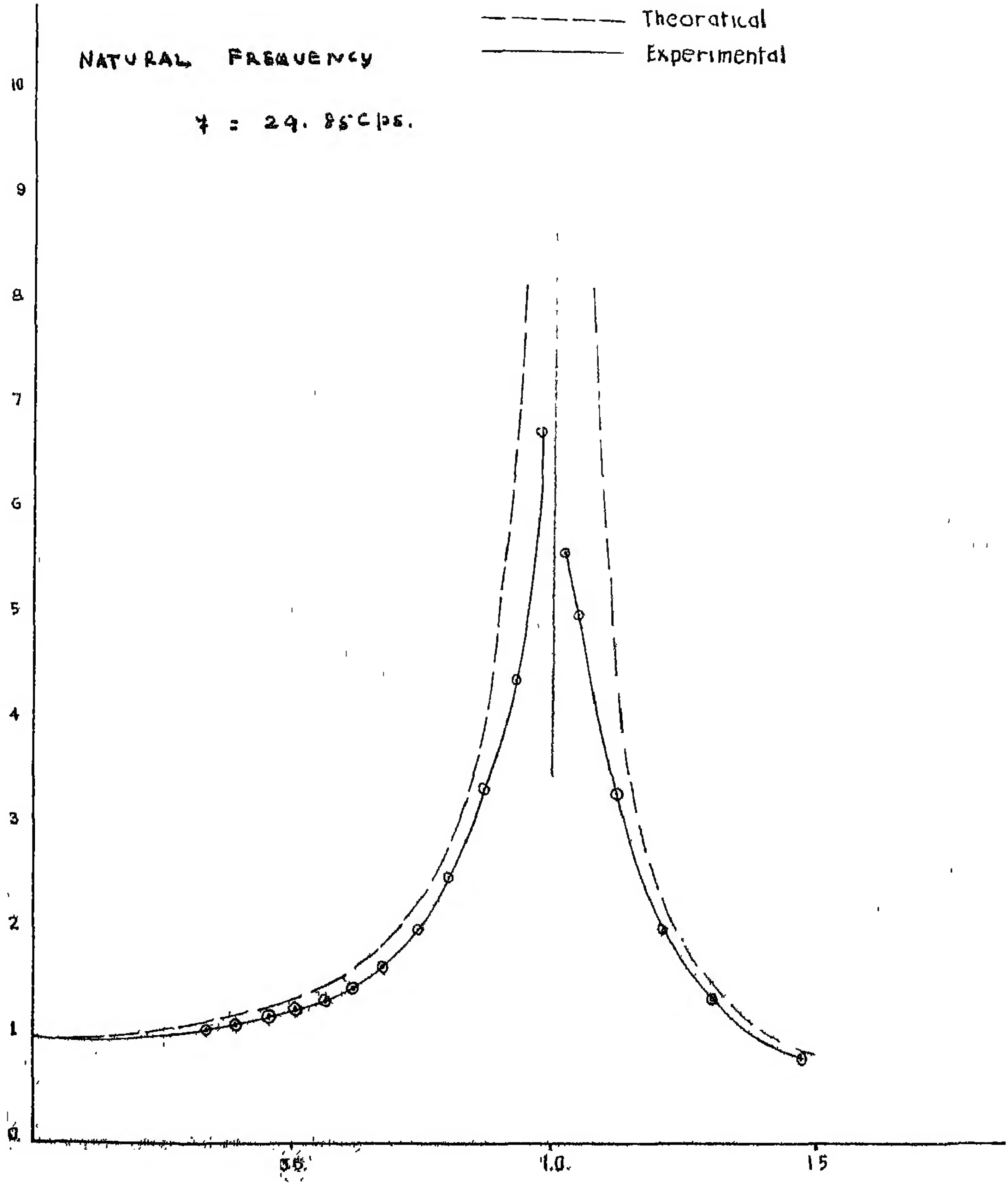


SINGLE STOREY

GRAPH-7

PLOT OF DISPLACEMENT/λ AGAINST FORCING FREQUENCY/ NATURAL FREQUENCY

FOR λ = 0 1 cm

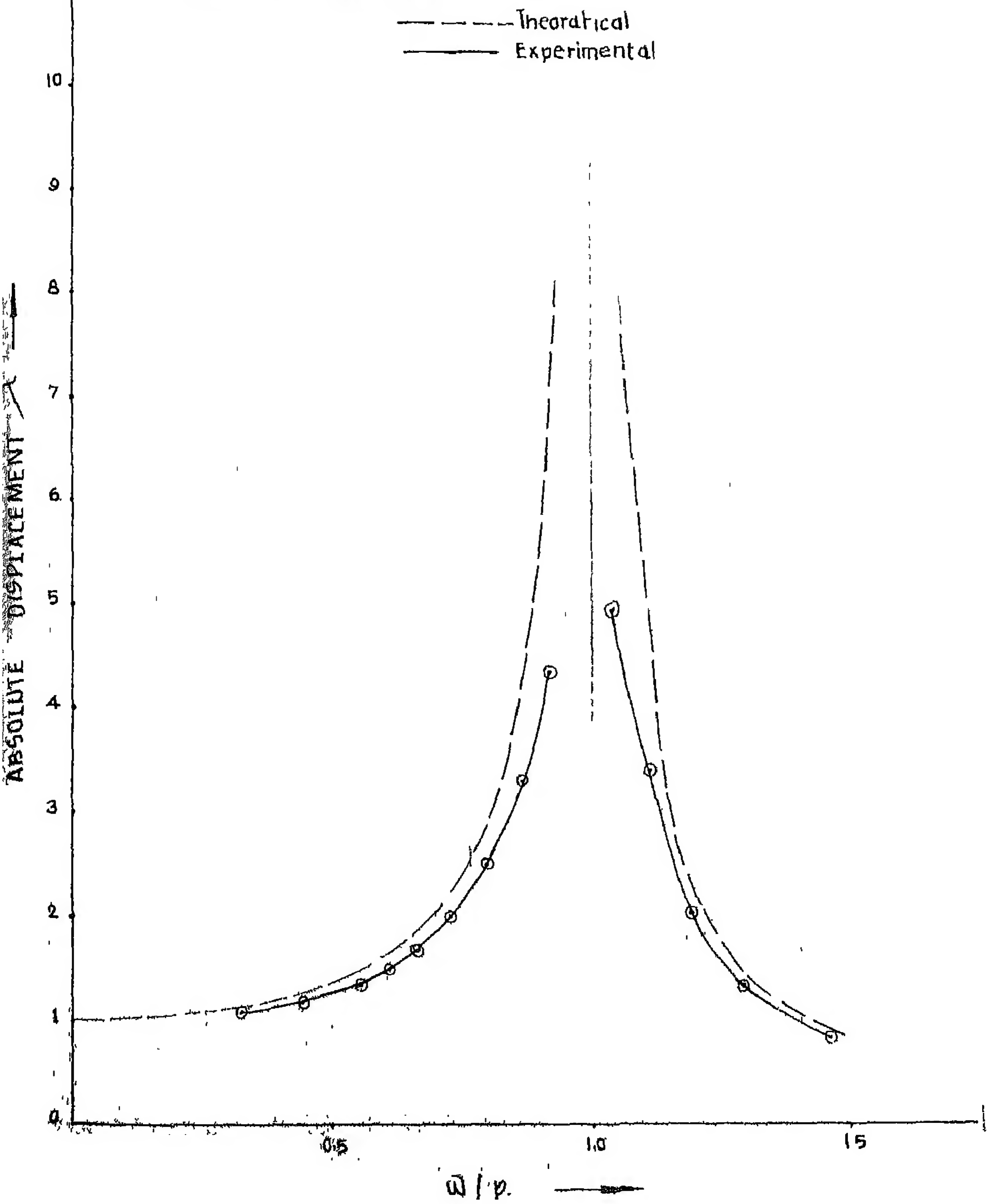


SINGLE STOREY

GRAPH-8

PLOT OF ABSOLUTE DISPLACEMENT / λ AGAINST FORCING FREQUENCY / NATURAL FREQUENCY

FOR λ = 0.15 cm

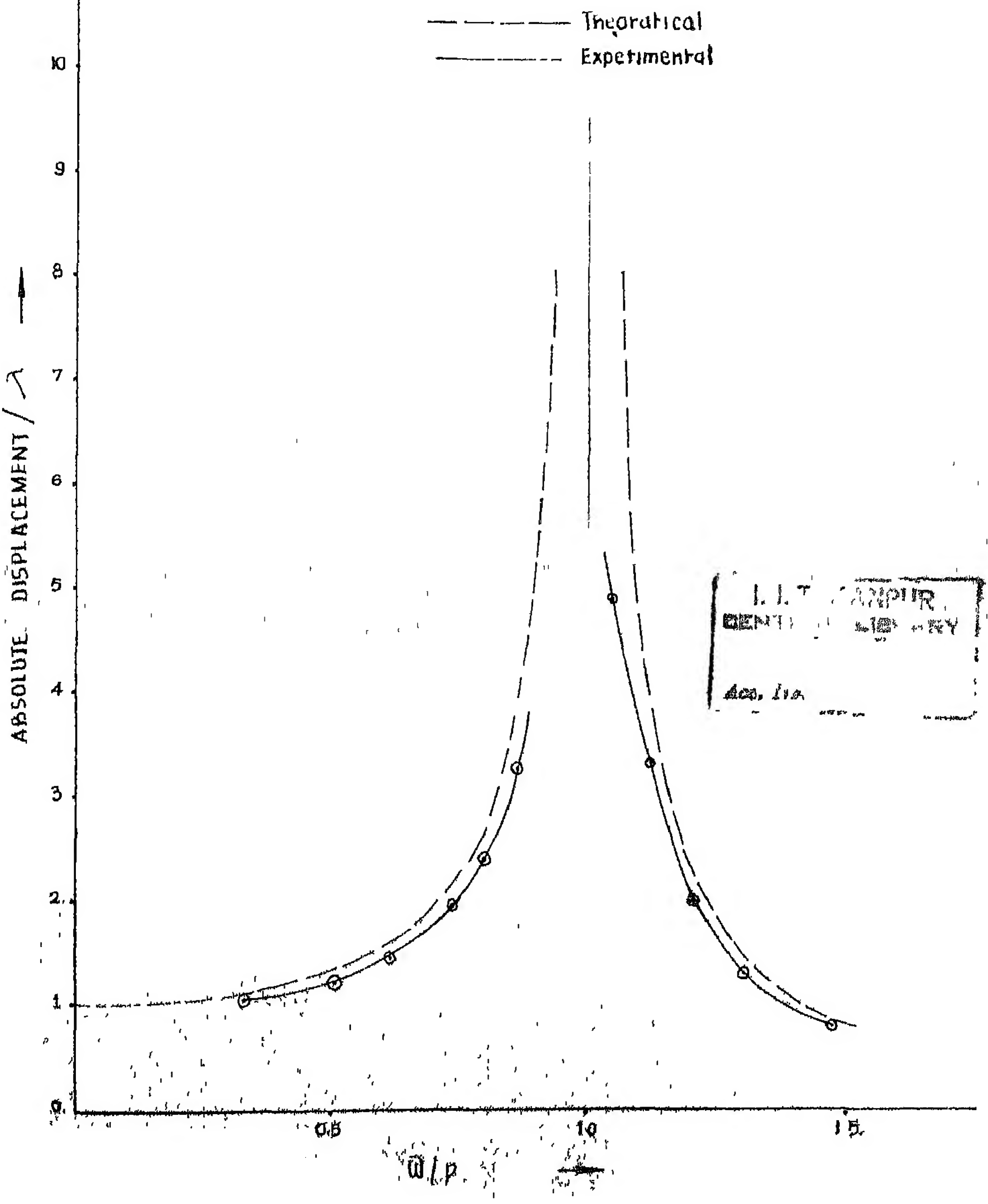


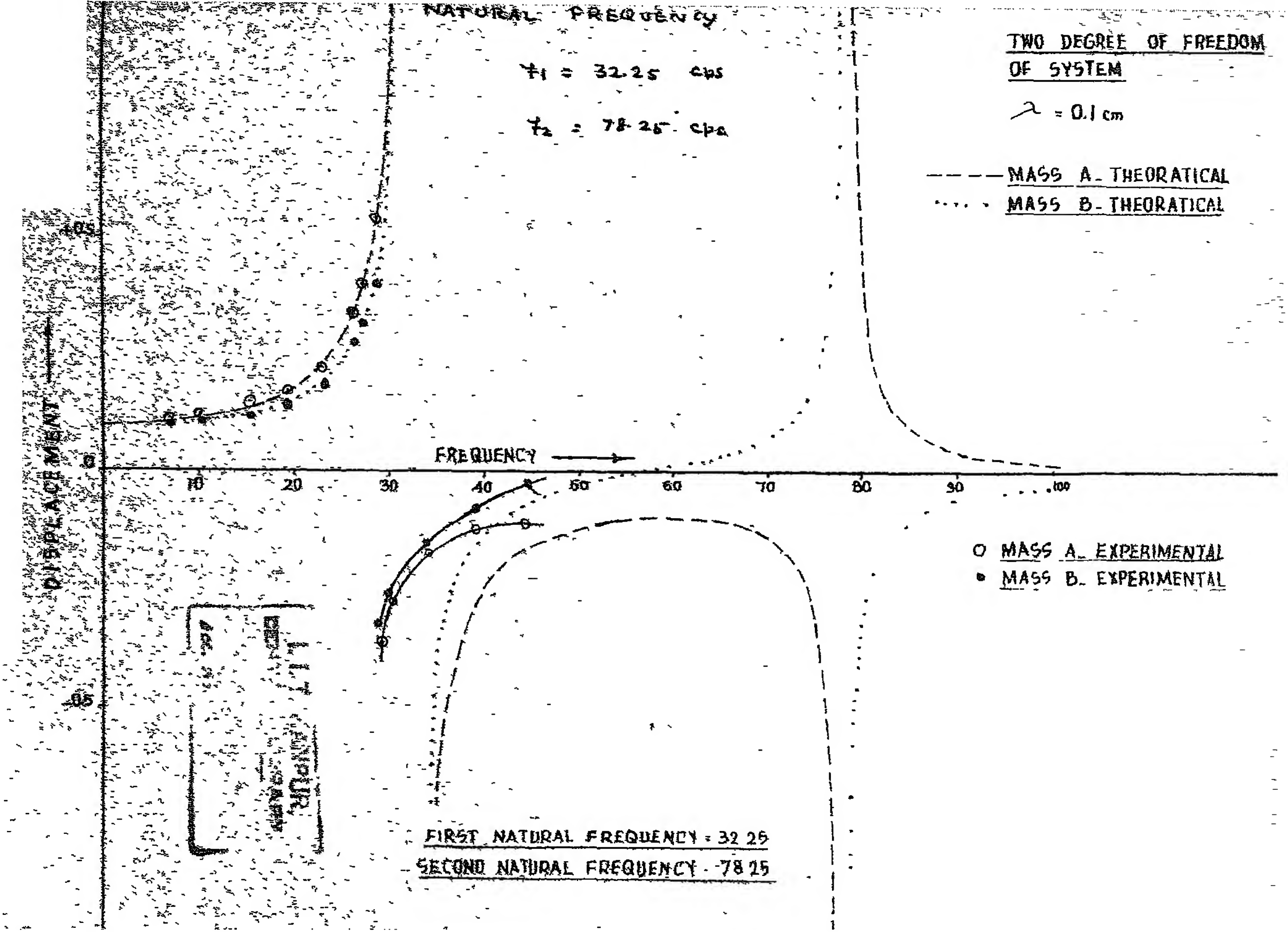
SINGLE STOREY

GRAPH-9

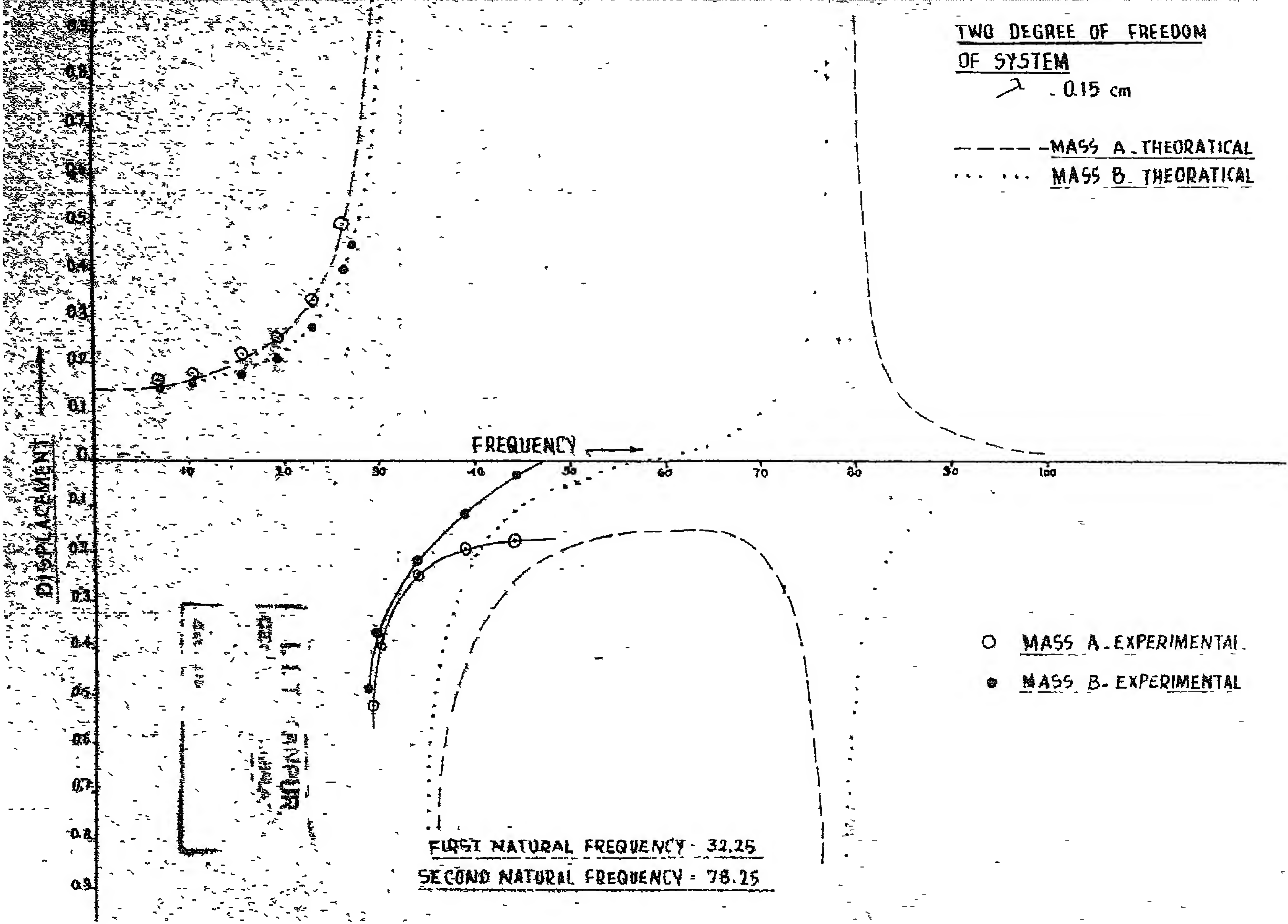
PLOT OF ABSOLUTE DISPLACEMENT/λ AGAINST FORCING FREQUENCY/NATURAL FREQUENCY

FOR λ = 0.2 cm





NATURAL FREQUENCY
f_1 = 32.25 cps
f_2 = 78.25 cps
TWO DEGREE OF FREEDOM OF SYSTEM
λ = 0.1 cm
MASS A. THEORATICAL
MASS B. THEORATICAL
O MASS A. EXPERIMENTAL
• MASS B. EXPERIMENTAL
FREQUENCY
DISPLACEMENT
.05
0
.05
10
20
30
40
50
60
70
80
90
100
FIRST NATURAL FREQUENCY = 32.25
SECOND NATURAL FREQUENCY - 78.25



TWO DEGREE OF FREEDOM OF SYSTEM
λ . 0.15 cm
MASS A THEORATICAL
MASS B. THEORATICAL
MASS A EXPERIMENTAL
MASS B. EXPERIMENTAL
FREQUENCY
DISPLACEMENT
FIRST NATURAL FREQUENCY - 32.25
SECOND NATURAL FREQUENCY = 78.25

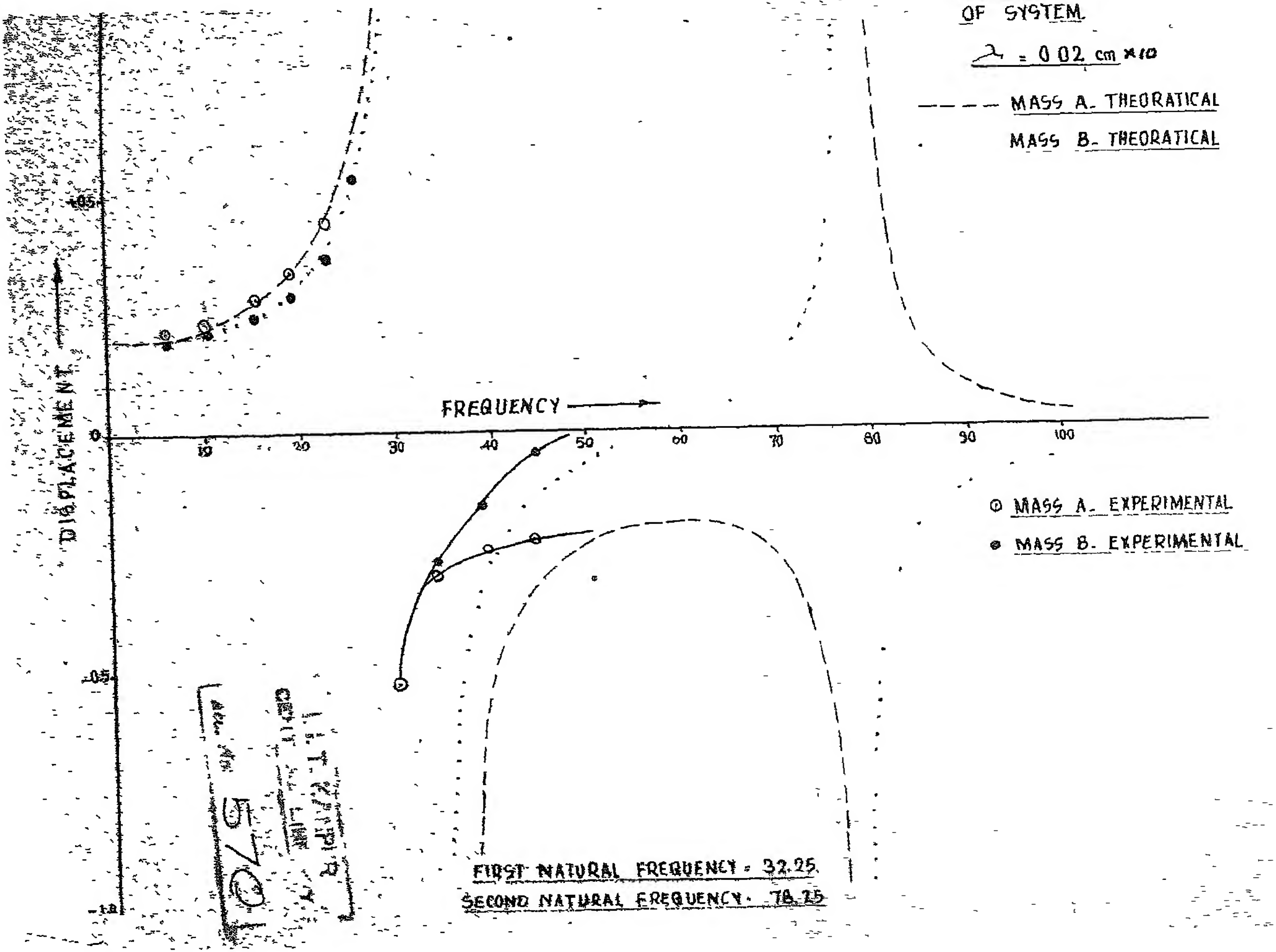

OF SYSTEM
λ = 0.02 cm ×10
MASS A. THEORATICAL
MASS B. THEORATICAL
MASS A. EXPERIMENTAL
MASS B. EXPERIMENTAL
FREQUENCY
DISPLACEMENT
0
10
20
30
40
50
60
70
80
90
100
.05
.05
1.0
FIRST NATURAL FREQUENCY = 32.25
SECOND NATURAL FREQUENCY - 78.25

PHOTO - 1

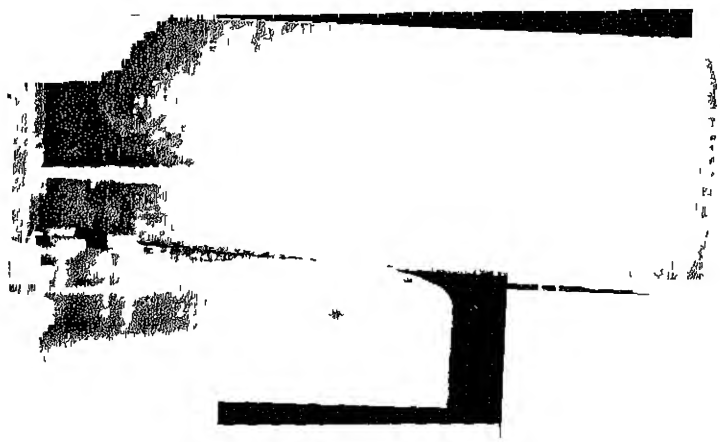


PHOTO - 2

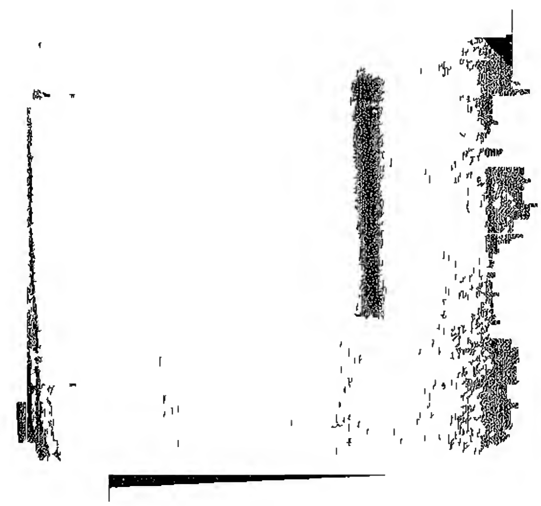


PHOTO - 3

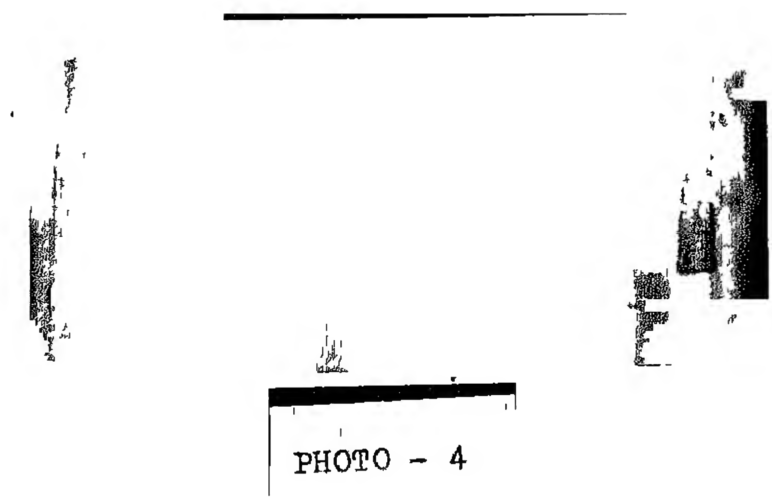


PHOTO - 4

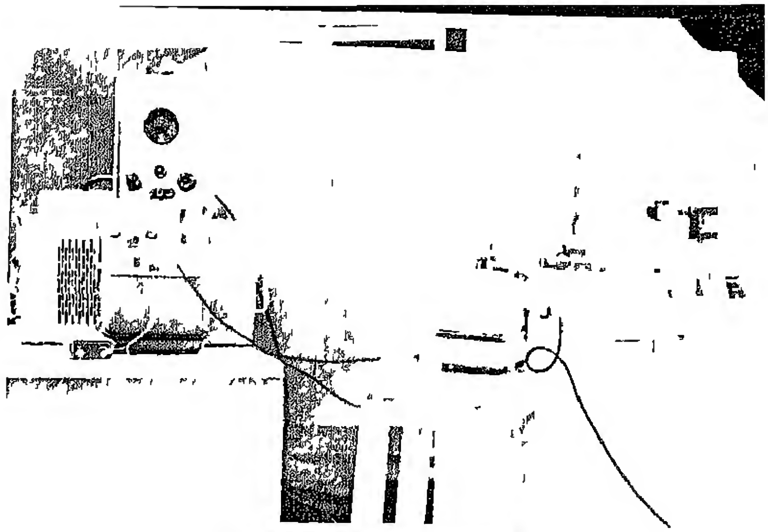


PHOTO - 5

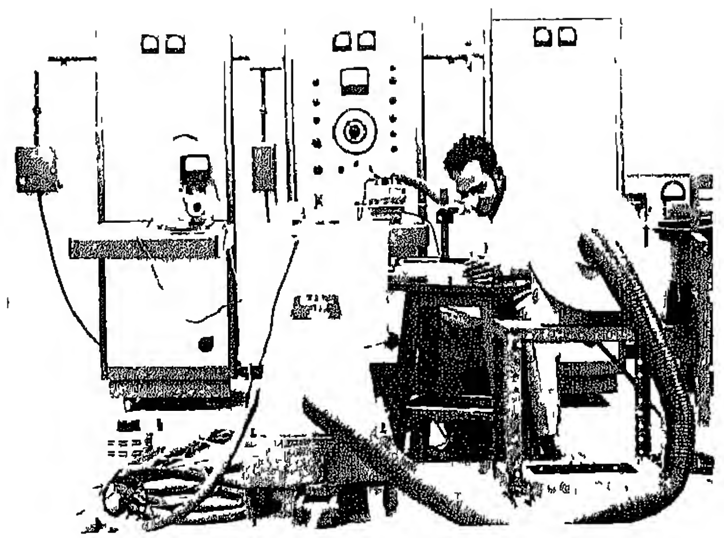


PHOTO - 6